The goal of physics is to provide an understanding of the physical world by developing theories based on experiments. A physical theory, usually expressed mathematically, describes how a given physical system works. The theory makes certain predictions about the physical system which can then be checked by observations and experiments. If the predictions turn out to correspond closely to what is actually observed, then the theory stands, although it remains provisional. No theory to date has given a complete description of all physical phenomena, even within a given subdiscipline of physics. Every theory is a work in progress.

The basic laws of physics involve such physical quantities as force, velocity, volume, and acceleration, all of which can be described in terms of more fundamental quantities. In mechanics, it is conventional to use the quantities of length (L), mass (M), and time (T); all other physical quantities can be constructed from these three.

### 1.1 Standards of Length, Mass, and Time

To communicate the result of a measurement of a certain physical quantity, a unit for the quantity must be defined. If our fundamental unit of length is defined to be 1.0 meter, for example, and someone familiar with our system of measurement reports that a wall is 2.0 meters high, we know that the height of the wall is twice the fundamental unit of length. Likewise, if our fundamental unit of mass is defined as 1.0 kilogram and we are told that a person has a mass of 75 kilograms, then that person has a mass 75 times as great as the fundamental unit of mass.

In 1960 an international committee agreed on a standard system of units for the fundamental quantities of science, called SI (Système International). Its units of length, mass, and time are the meter, kilogram, and second, respectively.

Stonehenge, in southern England, was built thousands of years ago to help keep track of the seasons. At dawn on the summer solstice the sun can be seen through these giant stone slabs.
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Length

In 1799 the legal standard of length in France became the meter, defined as one ten-millionth of the distance from the equator to the North Pole. Until 1960, the official length of the meter was the distance between two lines on a specific bar of platinum-iridium alloy stored under controlled conditions. This standard was abandoned for several reasons, the principal one being that measurements of the separation between the lines are not precise enough. In 1960 the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp. In October 1983 this definition was abandoned also, and the meter was redefined as the distance traveled by light in vacuum during a time interval of 1/299 792 458 second. This latest definition establishes the speed of light at 299 792 458 meters per second.

Mass

The SI unit of mass, the kilogram, is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France (similar to that shown in Fig. 1.1a). As we’ll see in Chapter 4, mass is a quantity used to measure the resistance to a change in the motion of an object. It’s more difficult to cause a change in the motion of an object with a large mass than an object with a small mass.

Time

Before 1960, the time standard was defined in terms of the average length of a solar day in the year 1900. (A solar day is the time between successive appearances of the Sun at the highest point it reaches in the sky each day.) The basic unit of time, the second, was defined to be \( \frac{1}{60} \times \frac{1}{60} \times \frac{1}{24} = \frac{1}{86 400} \) of the average solar day. In 1967 the second was redefined to take advantage of the high precision attainable with an atomic clock, which uses the characteristic frequency of the light emitted from the cesium-133 atom as its “reference clock.” The second is now defined as 9 192 631 700 times the period of oscillation of radiation from the cesium atom. The newest type of cesium atomic clock is shown in Figure 1.1b.

Tip 1.1 No Commas in Numbers with Many Digits

In science, numbers with more than three digits are written in groups of three digits separated by spaces rather than commas; so that 10 000 is the same as the common American notation 10,000. Similarly, \( \pi = 3.14159265 \) is written as 3.141 592 65.
Powers of 10 and their abbreviations are listed in Table 1.4. For example, $10^{-3}$ is defined as the time required for one complete vibration.

### Approximate Values for Length, Mass, and Time Intervals

Approximate values of some lengths, masses, and time intervals are presented in Tables 1.1, 1.2, and 1.3, respectively. Note the wide ranges of values. Study these tables to get a feel for a kilogram of mass (this book has a mass of about 2 kilograms), a time interval of $10^{10}$ seconds (one century is about $3 \times 10^5$ seconds), or two meters of length (the approximate height of a forward on a basketball team). Appendix A reviews the notation for powers of 10, such as the expression of the number 50,000 in the form $5 \times 10^4$.

Systems of units commonly used in physics are the Système International, in which the units of length, mass, and time are the meter (m), kilogram (kg), and second (s); the cgs, or Gaussian, system, in which the units of length, mass, and time are the centimeter (cm), gram (g), and second; and the U.S. customary system, in which the units of length, mass, and time are the foot (ft), slug, and second. SI units are almost universally accepted in science and industry, and will be used throughout the book. Limited use will be made of Gaussian and U.S. customary units.

Some of the most frequently used “metric” (SI and cgs) prefixes representing powers of 10 and their abbreviations are listed in Table 1.4. For example, $10^{-3}$ m is

### Table 1.1 Approximate Values of Some Measured Lengths

<table>
<thead>
<tr>
<th>Description</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from Earth to most remote known quasar</td>
<td>$1 \times 10^{26}$</td>
</tr>
<tr>
<td>Distance from Earth to most remote known normal galaxies</td>
<td>$4 \times 10^{25}$</td>
</tr>
<tr>
<td>Distance from Earth to nearest large galaxy (M31, the Andromeda galaxy)</td>
<td>$2 \times 10^{22}$</td>
</tr>
<tr>
<td>Distance from Earth to nearest star (Proxima Centauri)</td>
<td>$4 \times 10^{16}$</td>
</tr>
<tr>
<td>One light year</td>
<td>$9 \times 10^{15}$</td>
</tr>
<tr>
<td>Mean orbit radius of Earth about Sun</td>
<td>$2 \times 10^{11}$</td>
</tr>
<tr>
<td>Mean distance from Earth to Moon</td>
<td>$4 \times 10^{8}$</td>
</tr>
<tr>
<td>Mean radius of Earth</td>
<td>$6 \times 10^{6}$</td>
</tr>
<tr>
<td>Typical altitude of satellite orbiting Earth</td>
<td>$2 \times 10^{5}$</td>
</tr>
<tr>
<td>Length of football field</td>
<td>$9 \times 10^{1}$</td>
</tr>
<tr>
<td>Length of housefly</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>Size of smallest dust particles</td>
<td>$1 \times 10^{-4}$</td>
</tr>
<tr>
<td>Size of cells in most living organisms</td>
<td>$1 \times 10^{-5}$</td>
</tr>
<tr>
<td>Diameter of hydrogen atom</td>
<td>$1 \times 10^{-10}$</td>
</tr>
<tr>
<td>Diameter of atomic nucleus</td>
<td>$1 \times 10^{-14}$</td>
</tr>
<tr>
<td>Diameter of proton</td>
<td>$1 \times 10^{-15}$</td>
</tr>
</tbody>
</table>

### Table 1.2 Approximate Values of Some Masses

<table>
<thead>
<tr>
<th>Description</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observable Universe</td>
<td>$1 \times 10^{52}$</td>
</tr>
<tr>
<td>Milky Way galaxy</td>
<td>$7 \times 10^{41}$</td>
</tr>
<tr>
<td>Sun</td>
<td>$2 \times 10^{30}$</td>
</tr>
<tr>
<td>Earth</td>
<td>$6 \times 10^{24}$</td>
</tr>
<tr>
<td>Moon</td>
<td>$7 \times 10^{22}$</td>
</tr>
<tr>
<td>Shark</td>
<td>$1 \times 10^{2}$</td>
</tr>
<tr>
<td>Human</td>
<td>$7 \times 10^{1}$</td>
</tr>
<tr>
<td>Frog</td>
<td>$1 \times 10^{-1}$</td>
</tr>
<tr>
<td>Mosquito</td>
<td>$1 \times 10^{-5}$</td>
</tr>
<tr>
<td>Bacterium</td>
<td>$1 \times 10^{-15}$</td>
</tr>
<tr>
<td>Hydrogen atom</td>
<td>$2 \times 10^{-27}$</td>
</tr>
<tr>
<td>Electron</td>
<td>$9 \times 10^{-31}$</td>
</tr>
</tbody>
</table>

### Table 1.3 Approximate Values of Some Time Intervals

<table>
<thead>
<tr>
<th>Description</th>
<th>Time Interval (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of Universe</td>
<td>$5 \times 10^{17}$</td>
</tr>
<tr>
<td>Age of Earth</td>
<td>$1 \times 10^{17}$</td>
</tr>
<tr>
<td>Average age of college student</td>
<td>$6 \times 10^{8}$</td>
</tr>
<tr>
<td>One year</td>
<td>$3 \times 10^{7}$</td>
</tr>
<tr>
<td>One day</td>
<td>$9 \times 10^{4}$</td>
</tr>
<tr>
<td>Time between normal heartbeats</td>
<td>$8 \times 10^{-1}$</td>
</tr>
<tr>
<td>Perioda of audible sound waves</td>
<td>$1 \times 10^{-5}$</td>
</tr>
<tr>
<td>Perioda of typical radio waves</td>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Perioda of vibration of atom in solid</td>
<td>$1 \times 10^{-13}$</td>
</tr>
<tr>
<td>Perioda of visible light waves</td>
<td>$2 \times 10^{-15}$</td>
</tr>
<tr>
<td>Duration of nuclear collision</td>
<td>$1 \times 10^{-22}$</td>
</tr>
<tr>
<td>Time required for light to travel across a proton</td>
<td>$3 \times 10^{-24}$</td>
</tr>
</tbody>
</table>

*A period is defined as the time required for one complete vibration.

### Table 1.4 Some Prefixes for Powers of Ten Used with “Metric” (SI and cgs) Units

<table>
<thead>
<tr>
<th>Power</th>
<th>Prefix</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-18}$</td>
<td>atto-</td>
<td>a</td>
</tr>
<tr>
<td>$10^{-15}$</td>
<td>femto-</td>
<td>f</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>pico-</td>
<td>p</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>nano-</td>
<td>n</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>micro-</td>
<td>µ</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>milli-</td>
<td>m</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>centi-</td>
<td>c</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>deci-</td>
<td>d</td>
</tr>
<tr>
<td>$10^{1}$</td>
<td>deka-</td>
<td>da</td>
</tr>
<tr>
<td>$10^{3}$</td>
<td>kilo-</td>
<td>k</td>
</tr>
<tr>
<td>$10^{6}$</td>
<td>mega-</td>
<td>M</td>
</tr>
<tr>
<td>$10^{9}$</td>
<td>giga-</td>
<td>G</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>tera-</td>
<td>T</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>peta-</td>
<td>P</td>
</tr>
<tr>
<td>$10^{18}$</td>
<td>exa-</td>
<td>E</td>
</tr>
</tbody>
</table>
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Equivalent to 1 millimeter (mm), and \(10^3\) m is 1 kilometer (km). Likewise, 1 kg is equal to \(10^3\) g, and 1 megavolt (MV) is \(10^6\) volts (V). It’s a good idea to memorize the more common prefixes early on: femto- to centi-, and kilo- to giga- are used routinely by most physicists.

1.2 The Building Blocks of Matter

A 1-kg (≈ 2-lb) cube of solid gold has a length of about 3.73 cm (≈ 1.5 in.) on a side. If the cube is cut in half, the two resulting pieces retain their chemical identity. But what happens if the pieces of the cube are cut again and again, indefinitely? The Greek philosophers Leucippus and Democritus couldn’t accept the idea that such cutting could go on forever. They speculated that the process ultimately would end when it produced a particle that could no longer be cut. In Greek, \textit{atomos} means “not sliceable.” From this term comes our English word \textit{atom}, once believed to be the smallest particle of matter but since found to be a composite of more elementary particles.

The atom can be naively visualized as a miniature solar system, with a dense, positively charged nucleus occupying the position of the Sun and negatively charged electrons orbiting like planets. This model of the atom, first developed by the great Danish physicist Niels Bohr nearly a century ago, led to the understanding of certain properties of the simpler atoms such as hydrogen but failed to explain many fine details of atomic structure.

Notice the size of a hydrogen atom, listed in Table 1.1, and the size of a proton—the nucleus of a hydrogen atom—one hundred thousand times smaller. If the proton were the size of a Ping Pong ball, the electron would be a tiny speck about the size of a bacterium, orbiting the proton a kilometer away! Other atoms are similarly constructed. So there is a surprising amount of empty space in ordinary matter.

After the discovery of the nucleus in the early 1900s, questions arose concerning its structure. Although the structure of the nucleus remains an area of active research even today, by the early 1930s scientists determined that two basic entities—protons and neutrons—occupy the nucleus. The proton is nature’s most common carrier of positive charge, equal in magnitude but opposite in sign to the charge on the electron. The number of protons in a nucleus determines what the element is. For instance, a nucleus containing only one proton is the nucleus of an atom of hydrogen, regardless of how many neutrons may be present. Extra neutrons correspond to different isotopes of hydrogen—deuterium and tritium—which react chemically in exactly the same way as hydrogen, but are more massive. An atom having two protons in its nucleus, similarly, is always helium, although again, differing numbers of neutrons are possible.

The existence of neutrons was verified conclusively in 1932. A neutron has no charge and has a mass about equal to that of a proton. Except for hydrogen, all atomic nuclei contain neutrons, which, together with the protons, interact through the strong nuclear force. That force opposes the strongly repulsive electrical force of the protons, which otherwise would cause the nucleus to disintegrate.

The division doesn’t stop here; strong evidence over many years indicates that protons, neutrons, and a zoo of other exotic particles are composed of six particles called quarks (rhymes with “sharks” though some rhyme it with “forks”). These particles have been given the names \textit{up}, \textit{down}, \textit{strange}, \textit{charm}, \textit{bottom}, and \textit{top}. The up, charm, and top quarks each carry a charge equal to +\(\frac{2}{3}\) that of the proton, whereas the down, strange, and bottom quarks each carry a charge equal to −\(\frac{1}{3}\) the proton charge. The proton consists of two up quarks and one down quark (see Fig. 1.2), giving the correct charge for the proton, +1. The neutron is composed of two down quarks and one up quark and has a net charge of zero.
The up and down quarks are sufficient to describe all normal matter, so the existence of the other four quarks, indirectly observed in high-energy experiments, is something of a mystery. Despite strong indirect evidence, no isolated quark has ever been observed. Consequently, the possible existence of yet more fundamental particles remains purely speculative.

### 1.3 Dimensional Analysis

In physics the word *dimension* denotes the physical nature of a quantity. The distance between two points, for example, can be measured in feet, meters, or furlongs, which are different ways of expressing the dimension of *length*.

The symbols used in this section to specify the dimensions of length, mass, and time are \( L, M, \) and \( T \), respectively. Brackets \([\ ]\) will often be used to denote the dimensions of a physical quantity. In this notation, for example, the dimensions of velocity \( v \) are \([v] = L/T\), and the dimensions of area \( A \) are \([A] = L^2\). The dimensions of area, volume, velocity, and acceleration are listed in Table 1.5, along with their units in the three common systems. The dimensions of other quantities, such as force and energy, will be described later as they are introduced.

In physics it’s often necessary to deal with mathematical expressions that relate different physical quantities. One way to analyze such expressions, called *dimensional analysis*, makes use of the fact that *dimensions can be treated as algebraic quantities*. Adding masses to lengths, for example, makes no sense, so it follows that quantities can be added or subtracted only if they have the same dimensions. If the terms on the opposite sides of an equation have the same dimensions, then that equation may be correct, although correctness can’t be guaranteed on the basis of dimensions alone. Nonetheless, dimensional analysis has value as a partial check of an equation and can also be used to develop insight into the relationships between physical quantities.

The procedure can be illustrated by using it to develop some relationships between acceleration, velocity, time, and distance. Distance \( x \) has the dimension of length: \([x] = L\). Time \( t \) has dimension \([t] = T\). Velocity \( v \) has the dimensions length over time: \([v] = L/T\), and acceleration the dimensions length divided by time squared: \([a] = L/T^2\). Notice that velocity and acceleration have similar dimensions, except for an extra dimension of time in the denominator of acceleration. It follows that

\[
[v] = \frac{L}{T} = \frac{L}{T^2} T = [a][t]
\]

From this it might be guessed that velocity equals acceleration multiplied by time, \( v = at \), and that is true for the special case of motion with constant acceleration starting at rest. Noticing that velocity has dimensions of length divided by time and distance has dimensions of length, it’s reasonable to guess that

\[
[x] = L = L \frac{T}{T} = \frac{L}{T^2} T = [v][t] = [a][t]^2
\]

### Table 1.5 Dimensions and Some Units of Area, Volume, Velocity, and Acceleration

<table>
<thead>
<tr>
<th>System</th>
<th>Area (( L^2 ))</th>
<th>Volume (( L^3 ))</th>
<th>Velocity (( L/T ))</th>
<th>Acceleration (( L/T^2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td>( m^2 )</td>
<td>( m^3 )</td>
<td>( m/s )</td>
<td>( m/s^2 )</td>
</tr>
<tr>
<td>cgs</td>
<td>( cm^2 )</td>
<td>( cm^3 )</td>
<td>( cm/s )</td>
<td>( cm/s^2 )</td>
</tr>
<tr>
<td>U.S. customary</td>
<td>( ft^2 )</td>
<td>( ft^3 )</td>
<td>( ft/s )</td>
<td>( ft/s^2 )</td>
</tr>
</tbody>
</table>
Here it appears that \( x = at^2 \) might correctly relate the distance traveled to acceleration and time; however, that equation is not even correct in the case of constant acceleration starting from rest. The correct expression in that case is \( x = \frac{1}{2}at^2 \). These examples serve to show the limitations inherent in using dimensional analysis to discover relationships between physical quantities. Nonetheless, such simple procedures can still be of value in developing a preliminary mathematical model for a given physical system. Further, because it’s easy to make errors when solving problems, dimensional analysis can be used to check the consistency of the results. When the dimensions in an equation are not consistent, that indicates an error has been made in a prior step.

### Example 1.1 Analysis of an Equation

**Goal** Check an equation using dimensional analysis.

**Problem** Show that the expression \( v = v_0 + at \) is dimensionally correct, where \( v \) and \( v_0 \) represent velocities, \( a \) is acceleration, and \( t \) is a time interval.

**Strategy** Analyze each term, finding its dimensions, and then check to see if all the terms agree with each other.

**Solution**

Find dimensions for \( v \) and \( v_0 \):

\[
[v] = [v_0] = \frac{L}{T}
\]

Find the dimensions of \( at \):

\[
[at] = [a][t] = \frac{L}{T^2}(T) = \frac{L}{T}
\]

**Remarks** All the terms agree, so the equation is dimensionally correct.

**Question 1.1** True or False. An equation that is dimensionally correct is always physically correct, up to a constant of proportionality.

**Exercise 1.1** Determine whether the equation \( x = vt^2 \) is dimensionally correct. If not, provide a correct expression, up to an overall constant of proportionality.

**Answer** Incorrect. The expression \( x = vt \) is dimensionally correct.

### Example 1.2 Find an Equation

**Goal** Derive an equation by using dimensional analysis.

**Problem** Find a relationship between a constant acceleration \( a \), speed \( v \), and distance \( r \) from the origin for a particle traveling in a circle.

**Strategy** Start with the term having the most dimensionality, \( a \). Find its dimensions, and then rewrite those dimensions in terms of the dimensions of \( v \) and \( r \). The dimensions of time will have to be eliminated with \( v \), because that's the only quantity (other than \( a \), itself) in which the dimension of time appears.

**Solution**

Write down the dimensions of \( a \):

\[
[a] = \frac{L}{T^2}
\]

Solve the dimensions of speed for \( T \):

\[
[v] = \frac{L}{T} \quad \rightarrow \quad T = \frac{L}{[v]}
\]

Substitute the expression for \( T \) into the equation for \( [a] \):

\[
[a] = \frac{L}{T^2} = \frac{L}{(L/[v])^2} = \frac{[v]^2}{L}
\]

Substitute \( L = [r] \), and guess at the equation:

\[
[a] = \frac{[v]^2}{[r]} \quad \rightarrow \quad a = \frac{v^2}{r}
\]
1.4 Uncertainty in Measurement and Significant Figures

Physics is a science in which mathematical laws are tested by experiment. No physical quantity can be determined with complete accuracy because our senses are physically limited, even when extended with microscopes, cyclotrons, and other instruments. Consequently, it’s important to develop methods of determining the accuracy of measurements.

All measurements have uncertainties associated with them, whether or not they are explicitly stated. The accuracy of a measurement depends on the sensitivity of the apparatus, the skill of the person carrying out the measurement, and the number of times the measurement is repeated. Once the measurements, along with their uncertainties, are known, it’s often the case that calculations must be carried out using those measurements. Suppose two such measurements are multiplied. When a calculator is used to obtain this product, there may be eight digits in the calculator window, but often only two or three of those numbers have any significance. The rest have no value because they imply greater accuracy than was actually achieved in the original measurements. In experimental work, determining how many numbers to retain requires the application of statistics and the mathematical propagation of uncertainties. In a textbook it isn’t practical to apply those sophisticated tools in the numerous calculations, so instead a simple method, called significant figures, is used to indicate the approximate number of digits that should be retained at the end of a calculation. Although that method is not mathematically rigorous, it’s easy to apply and works fairly well.

Suppose that in a laboratory experiment we measure the area of a rectangular plate with a meter stick. Let’s assume that the accuracy to which we can measure a particular dimension of the plate is ±0.1 cm. If the length of the plate is measured to be 16.3 cm, we can claim only that it lies somewhere between 16.2 cm and 16.4 cm. In this case, we say that the measured value has three significant figures. Likewise, if the plate’s width is measured to be 4.5 cm, the actual value lies between 4.4 cm and 4.6 cm. This measured value has only two significant figures. We could write the measured values as 16.3 ± 0.1 cm and 4.5 ± 0.1 cm. In general, a significant figure is a reliably known digit (other than a zero used to locate a decimal point). Note that in each case, the final number has some uncertainty associated with it, and is therefore not 100% reliable. Despite the uncertainty, that number is retained and considered significant because it does convey some information.
Suppose we would like to find the area of the plate by multiplying the two measured values together. The final value can range between \((16.3 - 0.1 \text{ cm})(4.5 - 0.1 \text{ cm}) = (16.2 \text{ cm})(4.4 \text{ cm}) = 71.28 \text{ cm}^2\) and \((16.3 + 0.1 \text{ cm})(4.5 + 0.1 \text{ cm}) = (16.4 \text{ cm})(4.6 \text{ cm}) = 75.44 \text{ cm}^2\). Claiming to know anything about the hundredths place, or even the tenths place, doesn’t make any sense, because it’s clear we can’t even be certain of the units place, whether it’s the 1 in 71, the 5 in 75, or somewhere in between. The tenths and the hundredths places are clearly not significant. We have some information about the units place, so that number is significant. Multiplying the numbers at the middle of the uncertainty ranges gives \((16.3 \text{ cm})(4.5 \text{ cm}) = 73.35 \text{ cm}^2\), which is also in the middle of the area’s uncertainty range. Because the hundredths and tenths are not significant, we drop them and take the answer to be 73 cm\(^2\), with an uncertainty of ±2 cm\(^2\). Note that the answer has two significant figures, the same number of figures as the least accurately known quantity being multiplied, the 4.5-cm width.

Calculations as carried out in the preceding paragraph can indicate the proper number of significant figures, but those calculations are time-consuming. Instead, two rules of thumb can be applied. The first, concerning multiplication and division, is as follows: **In multiplying (dividing) two or more quantities, the number of significant figures in the final product (quotient) is the same as the number of significant figures in the least accurate of the factors being combined, where least accurate means having the lowest number of significant figures.**

To get the final number of significant figures, it’s usually necessary to do some rounding. If the last digit dropped is less than 5, simply drop the digit. If the last digit dropped is greater than or equal to 5, raise the last retained digit by one.\(^1\)

Zeros may or may not be significant figures. Zeros used to position the decimal point in such numbers as 0.03 and 0.0075 are not considered significant figures. Hence, 0.03 has one significant figure, and 0.0075 has two.

When zeros are placed after other digits in a whole number, there is a possibility of misinterpretation. For example, suppose the mass of an object is given as 1 500 g. This value is ambiguous, because we don’t know whether the last two zeros are being used to locate the decimal point or whether they represent significant figures in the measurement.

Using scientific notation to indicate the number of significant figures removes this ambiguity. In this case, we express the mass as \(1.5 \times 10^3 \text{ g}\) if there are two significant figures in the measured value, \(1.50 \times 10^3 \text{ g}\) if there are three significant figures, and \(1.500 \times 10^3 \text{ g}\) if there are four. Likewise, 0.000 15 is expressed in scientific notation as \(1.5 \times 10^{-4}\) if it has two significant figures or as \(1.50 \times 10^{-4}\) if it has three significant figures. The three zeros between the decimal point and the digit 1 in the number 0.000 15 are not counted as significant figures because they only locate the decimal point. In this book, **most of the numerical examples and end-of-chapter problems will yield answers having two or three significant figures.**

For addition and subtraction, it’s best to focus on the number of decimal places in the quantities involved rather than on the number of significant figures. **When numbers are added (subtracted), the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum (difference).** For example, if we wish to compute 123 (zero decimal places) + 5.35 (two decimal places), the answer is 128 (zero decimal places) and not 128.35. If we compute the sum 1 000 1 (four decimal places) + 0.00 3 (four decimal places) = 1 000.4, the result has the correct number of decimal places, namely four. Observe that the rules for multiplying significant figures don’t work here because the answer has five significant figures even though one of the terms in the sum, 0.00 3, has only one significant figure. Likewise, if we perform the subtraction

\(^1\)Some prefer to round to the nearest even digit when the last dropped digit is 5, which has the advantage of rounding 5 up half the time and down half the time. For example, 1.55 would round to 1.6, but 1.45 would round to 1.4. Because the final significant figure is only one representative of a range of values given by the uncertainty, this very slight refinement will not be used in this text.
1.002 − 0.998 = 0.004, the result has three decimal places because each term in the subtraction has three decimal places.

To show why this rule should hold, we return to the first example in which we added 123 and 5.35, and rewrite these numbers as 123.xxx and 5.35x. Digits written with an x are completely unknown and can be any digit from 0 to 9. Now we line up 123.xxx and 5.35x relative to the decimal point and perform the addition, using the rule that an unknown digit added to a known or unknown digit yields an unknown:

\[
\begin{align*}
123.xxx \\
+ 5.35x \\
\hline
128.xxx
\end{align*}
\]

The answer of 128.xxx means that we are justified only in keeping the number 128 because everything after the decimal point in the sum is actually unknown. The example shows that the controlling uncertainty is introduced into an addition or subtraction by the term with the smallest number of decimal places.

**EXAMPLE 1.3  Carpet Calculations**

**GOAL** Apply the rules for significant figures.

**PROBLEM** Several carpet installers make measurements for carpet installation in the different rooms of a restaurant, reporting their measurements with inconsistent accuracy, as compiled in Table 1.6. Compute the areas for (a) the banquet hall, (b) the meeting room, and (c) the dining room, taking into account significant figures. (d) What total area of carpet is required for these rooms?

**Table 1.6 Dimensions of Rooms in Example 1.3**

<table>
<thead>
<tr>
<th></th>
<th>Length (m)</th>
<th>Width (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banquet hall</td>
<td>14.71</td>
<td>7.46</td>
</tr>
<tr>
<td>Meeting room</td>
<td>4.822</td>
<td>5.1</td>
</tr>
<tr>
<td>Dining room</td>
<td>13.8</td>
<td>9</td>
</tr>
</tbody>
</table>

**STRATEGY** For the multiplication problems in parts (a)–(c), count the significant figures in each number. The smaller result is the number of significant figures in the answer. Part (d) requires a sum, where the area with the least accurately known decimal place determines the overall number of significant figures in the answer.

**SOLUTION**

(a) Compute the area of the banquet hall.

Count significant figures:

\[14.71 \text{ m} \rightarrow 4 \text{ significant figures} \]
\[7.46 \text{ m} \rightarrow 3 \text{ significant figures} \]

To find the area, multiply the numbers keeping only three digits:

\[14.71 \text{ m} \times 7.46 \text{ m} = 109.74 \text{ m}^2 \rightarrow 1.10 \times 10^2 \text{ m}^2 \]

(b) Compute the area of the meeting room.

Count significant figures:

\[4.822 \text{ m} \rightarrow 4 \text{ significant figures} \]
\[5.1 \text{ m} \rightarrow 2 \text{ significant figures} \]

To find the area, multiply the numbers keeping only two digits:

\[4.822 \text{ m} \times 5.1 \text{ m} = 24.59 \text{ m}^2 \rightarrow 25 \text{ m}^2 \]

(c) Compute the area of the dining room.

Count significant figures:

\[13.8 \text{ m} \rightarrow 3 \text{ significant figures} \]
\[9 \text{ m} \rightarrow 1 \text{ significant figure} \]

To find the area, multiply the numbers keeping only one digit:

\[13.8 \times 9 \text{ m} = 124.2 \text{ m}^2 \rightarrow 100 \text{ m}^2 \]

(Continued)
(d) Calculate the total area of carpet required, with the proper number of significant figures.

Sum all three answers without regard to significant figures: \[ 1.10 \times 10^2 \text{ m}^2 + 25 \text{ m}^2 + 100 \text{ m}^2 = 235 \text{ m}^2 \]

The least accurate number is 100 m\(^2\), with one significant figure in the hundred’s decimal place:

\[ 235 \text{ m}^2 \rightarrow 2 \times 10^2 \text{ m}^2 \]

**REMARKS** Notice that the final answer in part (d) has only one significant figure, in the hundredth’s place, resulting in an answer that had to be rounded down by a sizable fraction of its total value. That’s the consequence of having insufficient information. The value of 9 m, without any further information, represents a true value that could be anywhere in the interval [8.5 m, 9.5 m), all of which round to 9 when only one digit is retained.

**QUESTION 1.3** How would the final answer change if the width of the dining room were given as 9.0 m?

**EXERCISE 1.3** A ranch has two fenced rectangular areas. Area A has a length of 750 m and width 125 m, and area B has length 400 m and width 150 m. Find (a) area A, (b) area B, and (c) the total area, with attention to the rules of significant figures. Assume trailing zeros are not significant.

**ANSWERS** (a) \(9.4 \times 10^4\) m\(^2\) (b) \(6 \times 10^4\) m\(^2\) (c) \(1.5 \times 10^5\) m\(^2\)

In performing any calculation, especially one involving a number of steps, there will always be slight discrepancies introduced by both the rounding process and the algebraic order in which steps are carried out. For example, consider \(2.35 \times 5.89/1.57\). This computation can be performed in three different orders. First, we have \(2.35 \times 5.89 = 13.842\), which rounds to 13.8, followed by 13.8/1.57 = 8.789, rounding to 8.79. Second, 5.89/1.57 = 3.751, which rounds to 3.75, resulting in 2.35 \times 3.75 = 8.125, rounding to 8.81. Finally, 2.35/1.57 = 1.496 rounds to 1.50, and 1.50 \times 5.89 = 8.855 rounds to 8.84. So three different algebraic orders, following the rules of rounding, lead to answers of 8.79, 8.81, and 8.84, respectively. Such minor discrepancies are to be expected, because the last significant digit is only one representative from a range of possible values, depending on experimental uncertainty. To avoid such discrepancies, some carry one or more extra digits during the calculation, although it isn’t conceptually consistent to do so because those extra digits are not significant. As a practical matter, in the worked examples in this text, intermediate reported results will be rounded to the proper number of significant figures, and only those digits will be carried forward. In the problem sets, however, given data will usually be assumed accurate to two or three digits, even when there are trailing zeros. In solving the problems, the student should be aware that slight differences in rounding practices can result in answers varying from the text in the last significant digit, which is normal and not cause for concern. The method of significant figures has its limitations in determining accuracy, but it’s easy to apply. In experimental work, however, statistics and the mathematical propagation of uncertainty must be used to determine the accuracy of an experimental result.

### 1.5 Conversion of Units

Sometimes it’s necessary to convert units from one system to another. Conversion factors between the SI and U.S. customary systems for units of length are as follows:

\[ 1 \text{ mi} = 1609 \text{ m} = 1.609 \text{ km} \quad 1 \text{ ft} = 0.3048 \text{ m} = 30.48 \text{ cm} \]

\[ 1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \quad 1 \text{ in.} = 0.0254 \text{ m} = 2.54 \text{ cm} \]

A more extensive list of conversion factors can be found on the front endpapers of this book. In all the given conversion equations, the “1” on the left is assumed to have the same number of significant figures as the quantity given on the right of the equation.

Units can be treated as algebraic quantities that can “cancel” each other. We can make a fraction with the conversion that will cancel the units we don’t want,
and multiply that fraction by the quantity in question. For example, suppose we want to convert 15.0 in. to centimeters. Because 1 in. = 2.54 cm, we find that

\[ 15.0 \text{ in.} = 15.0 \text{ in.} \times \left( \frac{2.54 \text{ cm}}{1.00 \text{ in.}} \right) = 38.1 \text{ cm} \]

The next two examples show how to deal with problems involving more than one conversion and with powers.

---

**EXAMPLE 1.4 Pull Over, Buddy!**

**GOAL** Convert units using several conversion factors.

**PROBLEM** If a car is traveling at a speed of 28.0 m/s, is the driver exceeding the speed limit of 55.0 mi/h?

**STRATEGY** Meters must be converted to miles and seconds to hours, using the conversion factors listed on the front end-sheets of the book. Here, three factors will be used.

**SOLUTION**

Convert meters to miles:

\[ 28.0 \text{ m/s} = \left( \frac{28.0 \text{ m}}{1.00 \text{ s}} \right) \left( \frac{1.00 \text{ mi}}{1609 \text{ m}} \right) = 1.74 \times 10^{-2} \text{ mi/s} \]

Convert seconds to hours:

\[ 1.74 \times 10^{-2} \text{ mi/s} = \left( 1.74 \times 10^{-2} \text{ mi/s} \right) \left( \frac{60.0 \text{ s}}{1 \text{ min}} \right) \left( \frac{60.0 \text{ min}}{1 \text{ h}} \right) = 62.6 \text{ mi/h} \]

**REMARKS** The driver should slow down because he’s exceeding the speed limit.

**QUESTION 1.4** Repeat the conversion, using the relationship 1.00 m/s = 2.24 mi/h. Why is the answer slightly different?

**EXERCISE 1.4** Convert 152 mi/h to m/s.

**ANSWER** 67.9 m/s

---

**EXAMPLE 1.5 Press the Pedal to the Metal**

**GOAL** Convert a quantity featuring powers of a unit.

**PROBLEM** The traffic light turns green, and the driver of a high-performance car slams the accelerator to the floor. The accelerometer registers 22.0 m/s². Convert this reading to km/min².

**STRATEGY** Here we need one factor to convert meters to kilometers and another two factors to convert seconds squared to minutes squared.

**SOLUTION**

Multiply by the three factors:

\[ \frac{22.0 \text{ m/s}^2}{1.00 \text{ s}^2} \left( \frac{1.00 \text{ km}}{1.00 \times 10^3 \text{ m}} \right) \left( \frac{60.0 \text{ s}}{1 \text{ min}} \right)^2 = 79.2 \text{ km/min}^2 \]

**REMARKS** Notice that in each conversion factor the numerator equals the denominator when units are taken into account. A common error in dealing with squares is to square the units inside the parentheses while forgetting to square the numbers!

**QUESTION 1.5** What time conversion factor would be used to further convert the answer to km/h²?

**EXERCISE 1.5** Convert 4.50 \( \times 10^3 \) kg/m³ to g/cm³.

**ANSWER** 4.50 g/cm³
1.6 Estimates and Order-of-Magnitude Calculations

Getting an exact answer to a calculation may often be difficult or impossible, either for mathematical reasons or because limited information is available. In these cases, estimates can yield useful approximate answers that can determine whether a more precise calculation is necessary. Estimates also serve as a partial check if the exact calculations are actually carried out. If a large answer is expected but a small exact answer is obtained, there’s an error somewhere.

For many problems, knowing the approximate value of a quantity—within a factor of 10 or so—is sufficient. This approximate value is called an order-of-magnitude estimate, and requires finding the power of 10 that is closest to the actual value of the quantity. For example, 75 kg \( \sim 10^2 \) kg, where the symbol \( \sim \) means “is on the order of” or “is approximately.” Increasing a quantity by three orders of magnitude means that its value increases by a factor of \( 10^3 = 1000 \).

Occasionally the process of making such estimates results in fairly crude answers, but answers ten times or more too large or small are still useful. For example, suppose you’re interested in how many people have contracted a certain disease. Any estimates under ten thousand are small compared with Earth’s total population, but a million or more would be alarming. So even relatively imprecise information can provide valuable guidance.

In developing these estimates, you can take considerable liberties with the numbers. For example, \( \pi \sim 1, 27 \sim 10, \) and \( 65 \sim 100 \). To get a less crude estimate, it’s permissible to use slightly more accurate numbers (e.g., \( \pi \sim 3, 27 \sim 30, 65 \sim 70 \)). Better accuracy can also be obtained by systematically underestimating as many numbers as you overestimate. Some quantities may be completely unknown, but it’s standard to make reasonable guesses, as the examples show.

---

### Example 1.6 Brain Cells Estimate

**Goal** Develop a simple estimate.

**Problem** Estimate the number of cells in the human brain.

**Strategy** Estimate the volume of a human brain and divide by the estimated volume of one cell. The brain is located in the upper portion of the head, with a volume that could be approximated by a cube \( \ell = 20 \text{ cm} \) on a side.

**Solution**

Estimate of the volume of a human brain:

\[
V_{\text{brain}} = \ell^3 \approx (0.2 \text{ m})^3 = 8 \times 10^{-3} \text{ m}^3 = 1 \times 10^{-2} \text{ m}^3
\]

Estimate the volume of a cell:

\[
V_{\text{cell}} = d^3 = (10 \times 10^{-6} \text{ m})^3 = 1 \times 10^{-15} \text{ m}^3
\]

Divide the volume of a brain by the volume of a cell:

\[
\text{number of cells} = \frac{V_{\text{brain}}}{V_{\text{cell}}} = \frac{0.01 \text{ m}^3}{1 \times 10^{-15} \text{ m}^3} = 1 \times 10^{13} \text{ cells}
\]

**Remarks** Notice how little attention was paid to obtaining precise values. Some general information about a problem is required if the estimate is to be within an order of magnitude of the actual value. Here, knowledge of the approximate dimensions of brain cells and the human brain were essential to developing the estimate.

**Question 1.6** Would \( 10^{12} \) cells also be a reasonable estimate? What about \( 10^9 \) cells? Explain.

**Exercise 1.6** Estimate the total number of cells in the human body.

**Answer** \( 10^{14} \) (Answers may vary.)
EXAMPLE 1.7  Stack One-Dollar Bills to the Moon

GOAL  Estimate the number of stacked objects required to reach a given height.

PROBLEM  How many one-dollar bills, stacked one on top of the other, would reach the Moon?

STRATEGY  The distance to the Moon is about 400 000 km. Guess at the number of dollar bills in a millimeter, and multiply the distance by this number, after converting to consistent units.

SOLUTION  We estimate that ten stacked bills form a layer of 1 mm. Convert mm to km:

\[
\frac{10 \text{ bills}}{1 \text{ mm}} \cdot \frac{10^3 \text{ mm}}{1 \text{ m}} \cdot \frac{1 \text{ km}}{10^3 \text{ m}} = \frac{10^7 \text{ bills}}{1 \text{ km}}
\]

Multiply this value by the approximate lunar distance:

\[
\text{# of dollar bills} \sim (4 \times 10^5 \text{ km}) \cdot \frac{10^7 \text{ bills}}{1 \text{ km}} = 4 \times 10^{12} \text{ bills}
\]

REMARKS  That’s within an order of magnitude of the U.S. national debt!

QUESTION 1.7  Based on the answer, about how many stacked pennies would reach the Moon?

EXERCISE 1.7  How many pieces of cardboard, typically found at the back of a bound pad of paper, would you have to stack up to match the height of the Washington monument, about 170 m tall?

ANSWER  \( \sim 10^5 \) (Answers may vary.)

EXAMPLE 1.8  Number of Galaxies in the Universe

GOAL  Estimate a volume and a number density, and combine.

PROBLEM  Given that astronomers can see about 10 billion light years into space and that there are 14 galaxies in our local group, 2 million light years from the next local group, estimate the number of galaxies in the observable universe. (Note: One light year is the distance traveled by light in one year, about \( 9.5 \times 10^{15} \text{ m} \).) (See Fig. 1.3.)

STRATEGY  From the known information, we can estimate the number of galaxies per unit volume. The local group of 14 galaxies is contained in a sphere a million light years in radius, with the Andromeda group in a similar sphere, so there are about 10 galaxies within a volume of radius 1 million light years. Multiply that number density by the volume of the observable universe.

SOLUTION  Compute the approximate volume \( V_{lg} \) of the local group of galaxies:

\[
V_{lg} = \frac{4}{3} \pi r^3 \sim (10^6 \text{ ly})^3 = 10^{18} \text{ ly}^3
\]

Estimate the density of galaxies:

\[
\text{density of galaxies} = \frac{\text{# of galaxies}}{V_{lg}} \sim \frac{10 \text{ galaxies}}{10^{18} \text{ ly}^3} = 10^{-17} \text{ galaxies/ly}^3
\]

Compute the approximate volume of the observable universe:

\[
V_u = \frac{4}{3} \pi r^3 \sim (10^{10} \text{ ly})^3 = 10^{30} \text{ ly}^3
\]

Multiply the density of galaxies by \( V_u \):

\[
\text{# of galaxies} \sim (\text{density of galaxies})V_u = \left(10^{-17} \frac{\text{galaxies}}{\text{ly}^3}\right)(10^{30} \text{ ly}^3) = 10^{13} \text{ galaxies}
\]

(Continued)
REMARKS Notice the approximate nature of the computation, which uses $4\pi/3 \approx 1$ on two occasions and $14 \sim 10$ for the number of galaxies in the local group. This is completely justified: Using the actual numbers would be pointless, because the other assumptions in the problem—the size of the observable universe and the idea that the local galaxy density is representative of the density everywhere—are also very rough approximations. Further, there was nothing in the problem that required using volumes of spheres rather than volumes of cubes. Despite all these arbitrary choices, the answer still gives useful information, because it rules out a lot of reasonable possible answers. Before doing the calculation, a guess of a billion galaxies might have seemed plausible.

QUESTION 1.8 About one in ten galaxies in the local group are not dwarf galaxies. Estimate the number of galaxies in the universe that are not dwarfs.

EXERCISE 1.8 (a) Given that the nearest star is about 4 light years away, develop an estimate of the density of stars per cubic light year in our galaxy. (b) Estimate the number of stars in the Milky Way galaxy, given that it’s roughly a disk 100,000 light years across and a thousand light years thick.

ANSWER (a) 0.02 stars/ly$^3$ (b) $2 \times 10^{11}$ stars (Estimates will vary. The actual answer is probably about twice that number.)

1.7 Coordinate Systems

Many aspects of physics deal with locations in space, which require the definition of a coordinate system. A point on a line can be located with one coordinate, a point in a plane with two coordinates, and a point in space with three.

A coordinate system used to specify locations in space consists of the following:

- A fixed reference point $O$, called the origin
- A set of specified axes, or directions, with an appropriate scale and labels on the axes
- Instructions on labeling a point in space relative to the origin and axes

One convenient and commonly used coordinate system is the Cartesian coordinate system, sometimes called the rectangular coordinate system. Such a system in two dimensions is illustrated in Figure 1.4. An arbitrary point in this system is labeled with the coordinates $(x, y)$. For example, the point $P$ in the figure has coordinates $(5, 3)$. If we start at the origin $O$, we can reach $P$ by moving 5 meters horizontally to the right and then 3 meters vertically upward. In the same way, the point $Q$ has coordinates $(-3, 4)$, which corresponds to going 3 meters horizontally to the left of the origin and 4 meters vertically upward from there.

Positive $x$ is usually selected as right of the origin and positive $y$ upward from the origin, but in two dimensions this choice is largely a matter of taste. (In three dimensions, however, there are “right-handed” and “left-handed” coordinates, which lead to minus sign differences in certain operations. These will be addressed as needed.)

Sometimes it’s more convenient to locate a point in space by its plane polar coordinates $(r, \theta)$, as in Figure 1.5. In this coordinate system, an origin $O$ and a reference line are selected as shown. A point is then specified by the distance $r$ from the origin to the point and by the angle $\theta$ between the reference line and a line drawn from the origin to the point. The standard reference line is usually selected to be the positive $x$-axis of a Cartesian coordinate system. The angle $\theta$ is considered positive when measured counterclockwise from the reference line and negative when measured clockwise. For example, if a point is specified by the polar coordinates $3$ m and $60^\circ$, we locate this point by moving out 3 m from the origin at an angle of $60^\circ$ above (counterclockwise from) the reference line. A point specified by polar coordinates $3$ m and $-60^\circ$ is located 3 m out from the origin and $60^\circ$ below (clockwise from) the reference line.
1.8 Trigonometry

Consider the right triangle shown in Active Figure 1.6, where side $y$ is opposite the angle $\theta$, side $x$ is adjacent to the angle $\theta$, and side $r$ is the hypotenuse of the triangle. The basic trigonometric functions defined by such a triangle are the ratios of the lengths of the sides of the triangle. These relationships are called the sine (sin), cosine (cos), and tangent (tan) functions. In terms of $\theta$, the basic trigonometric functions are as follows:

\[
\sin \theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{y}{r} \\
\cos \theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{x}{r} \\
\tan \theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{y}{x}
\]  \[1.1\]

For example, if the angle $\theta$ is equal to 30°, then the ratio of $y$ to $r$ is always 0.50; that is, $\sin 30° = 0.50$. Note that the sine, cosine, and tangent functions are quantities without units because each represents the ratio of two lengths.

Another important relationship, called the Pythagorean theorem, exists between the lengths of the sides of a right triangle:

\[r^2 = x^2 + y^2\]  \[1.2\]

Finally, it will often be necessary to find the values of inverse relationships. For example, suppose you know that the sine of an angle is 0.866, but you need to know the value of the angle itself. The inverse sine function may be expressed as $\sin^{-1} (0.866)$, which is a shorthand way of asking the question “What angle has a sine of 0.866?” Punching a couple of buttons on your calculator reveals that this angle is 60.0°. Try it for yourself and show that $\tan^{-1} (0.400) = 21.8°$. Be sure that your calculator is set for degrees and not radians. In addition, the inverse tangent function can return only values between $-90°$ and $90°$, so when an angle is in the second or third quadrant, it’s necessary to add 180° to the answer in the calculator window.

The definitions of the trigonometric functions and the inverse trigonometric functions, as well as the Pythagorean theorem, can be applied to any right triangle, regardless of whether its sides correspond to $x$- and $y$-coordinates.

These results from trigonometry are useful in converting from rectangular coordinates to polar coordinates, or vice versa, as the next example shows.

**EXAMPLE 1.9** Cartesian and Polar Coordinates

**GOAL** Understand how to convert from plane rectangular coordinates to plane polar coordinates and vice versa.

**PROBLEM** (a) The Cartesian coordinates of a point in the $xy$-plane are $(x, y) = (–3.50 \text{ m}, –2.50 \text{ m})$, as shown in Active Figure 1.7. Find the polar coordinates of this point. (b) Convert $(r, \theta) = (5.00 \text{ m}, 37.0°)$ to rectangular coordinates.

**STRATEGY** Apply the trigonometric functions and their inverses, together with the Pythagorean theorem.

---

Many people use the mnemonic SOHCAHTOA to remember the basic trigonometric formulas: Sine = Opposite/Hypotenuse, Cosine = Adjacent/Hypotenuse, and Tangent = Opposite/Adjacent. (Thanks go to Professor Don Chodrow for pointing this out.)
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Introduction

SOLUTION
(a) Cartesian to Polar conversion
Take the square root of both sides of Equation 1.2 to find the radial coordinate:

\[ r = \sqrt{x^2 + y^2} = \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2} = 4.30 \text{ m} \]

Use Equation 1.1 for the tangent function to find the angle with the inverse tangent, adding 180° because the angle is actually in third quadrant:

\[ \tan \theta = \frac{y}{x} = \frac{-2.50 \text{ m}}{-3.50 \text{ m}} = 0.714 \]

\[ \theta = \tan^{-1}(0.714) = 35.5° + 180° = 216° \]

(b) Polar to Cartesian conversion
Use the trigonometric definitions, Equation 1.1.

\[ x = r \cos \theta = (5.00 \text{ m}) \cos 37.0° = 3.99 \text{ m} \]

\[ y = r \sin \theta = (5.00 \text{ m}) \sin 37.0° = 3.01 \text{ m} \]

REMARKS When we take up vectors in two dimensions in Chapter 3, we will routinely use a similar process to find the direction and magnitude of a given vector from its components, or, conversely, to find the components from the vector’s magnitude and direction.

QUESTION 1.9 Starting with the answers to part (b), work backwards to recover the given radius and angle. Why are there slight differences from the original quantities?

EXERCISE 1.9 (a) Find the polar coordinates corresponding to \((x, y) = (-3.25 \text{ m}, 1.50 \text{ m})\). (b) Find the Cartesian coordinates corresponding to \((r, \theta) = (4.00 \text{ m}, 53.0°)\)

ANSWERS (a) \((r, \theta) = (3.58 \text{ m}, 155°)\) (b) \((x, y) = (2.41 \text{ m}, 3.19 \text{ m})\)

EXAMPLE 1.10 How High Is the Building?

GOAL Apply basic results of trigonometry.

PROBLEM A person measures the height of a building by walking out a distance of 46.0 m from its base and shining a flashlight beam toward the top. When the beam is elevated at an angle of 39.0° with respect to the horizontal, as shown in Figure 1.8, the beam just strikes the top of the building. (a) If the flashlight is held at a height of 2.00 m, find the height of the building. (b) Calculate the length of the light beam.

STRATEGY Refer to the right triangle shown in the figure. We know the angle, 39.0°, and the length of the side adjacent to it. Because the height of the building is the side opposite the angle, we can use the tangent function. With the adjacent and opposite sides known, we can then find the hypotenuse with the Pythagorean theorem.

SOLUTION (a) Find the height of the building.

Use the tangent of the given angle:

\[ \tan 39.0° = \frac{\Delta y}{46.0 \text{ m}} \]

Solve for the height:

\[ \Delta y = (\tan 39.0°)(46.0 \text{ m}) = (0.810)(46.0 \text{ m}) = 37.3 \text{ m} \]

Add 2.00 m to \(\Delta y\) to obtain the height:

height = 39.3 m

(b) Calculate the length of the light beam.

Use the Pythagorean theorem:

\[ r = \sqrt{x^2 + y^2} = \sqrt{(37.3 \text{ m})^2 + (46.0 \text{ m})^2} = 59.2 \text{ m} \]

REMARKS In a later chapter, right-triangle trigonometry is often used when working with vectors.

QUESTION 1.10 Could the distance traveled by the light beam be found without using the Pythagorean theorem? How?
EXERCISE 1.10 While standing atop a building 50.0 m tall, you spot a friend standing on a street corner. Using a protractor and dangling a plumb bob, you find that the angle between the horizontal and the direction to the spot on the sidewalk where your friend is standing is 25.0°. Your eyes are located 1.75 m above the top of the building. How far away from the foot of the building is your friend?

ANSWER 111 m

1.9 Problem-Solving Strategy

Most courses in general physics require the student to learn the skills used in solving problems, and examinations usually include problems that test such skills. This brief section presents some useful suggestions that will help increase your success in solving problems. An organized approach to problem solving will also enhance your understanding of physical concepts and reduce exam stress. Throughout the book, there will be a number of sections labeled “Problem-Solving Strategy,” many of them just a specializing of the list given below (and illustrated in Fig. 1.9).

General Problem-Solving Strategy

Problem

1. Read the problem carefully at least twice. Be sure you understand the nature of the problem before proceeding further.
2. Draw a diagram while rereading the problem.
3. Label all physical quantities in the diagram, using letters that remind you what the quantity is (e.g., \( m \) for mass). Choose a coordinate system and label it.

Strategy

4. Identify physical principles, the knowns and unknowns, and list them. Put circles around the unknowns. There must be as many equations as there are unknowns.
5. Equations, the relationships between the labeled physical quantities, should be written down next. Naturally, the selected equations should be consistent with the physical principles identified in the previous step.

Solution

6. Solve the set of equations for the unknown quantities in terms of the known. Do this algebraically, without substituting values until the next step, except where terms are zero.
7. Substitute the known values, together with their units. Obtain a numerical value with units for each unknown.

Check Answer

8. Check your answer. Do the units match? Is the answer reasonable? Does the plus or minus sign make sense? Is your answer consistent with an order of magnitude estimate?

This same procedure, with minor variations, should be followed throughout the course. The first three steps are extremely important, because they get you mentally oriented. Identifying the proper concepts and physical principles assists you in choosing the correct equations. The equations themselves are essential, because when you understand them, you also understand the relationships between the physical quantities. This understanding comes through a lot of daily practice.

Equations are the tools of physics: To solve problems, you have to have them at hand, like a plumber and his wrenches. Know the equations, and understand...
what they mean and how to use them. Just as you can’t have a conversation without knowing the local language, you can’t solve physics problems without knowing and understanding the equations. This understanding grows as you study and apply the concepts and the equations relating them.

Carrying through the algebra for as long as possible, substituting numbers only at the end, is also important, because it helps you think in terms of the physical quantities involved, not merely the numbers that represent them. Many beginning physics students are eager to substitute, but once numbers are substituted it’s harder to understand relationships and easier to make mistakes.

The physical layout and organization of your work will make the final product more understandable and easier to follow. Although physics is a challenging discipline, your chances of success are excellent if you maintain a positive attitude and keep trying.

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**Tip 1.4 Get Used to Symbolic Algebra**

Whenever possible, solve problems symbolically and then substitute known values. This process helps prevent errors and clarifies the relationships between physical quantities.

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**EXAMPLE 1.11 A Round Trip by Air**

**GOAL** Illustrate the Problem-Solving Strategy.

**PROBLEM** An airplane travels $x = 4.50 \times 10^2$ km due east and then travels an unknown distance $y$ due north. Finally, it returns to its starting point by traveling a distance of $r = 525$ km. How far did the airplane travel in the northerly direction?

**STRATEGY** We’ve finished reading the problem (step 1), and have drawn a diagram (step 2) in Figure 1.10 and labeled it (step 3). From the diagram, we recognize a right triangle and identify (step 4) the principle involved: the Pythagorean theorem. Side $y$ is the unknown quantity, and the other sides are known.

**SOLUTION** Write the Pythagorean theorem (step 5):

$$r^2 = x^2 + y^2$$

Solve symbolically for $y$ (step 6):

$$y^2 = r^2 - x^2 \quad \rightarrow \quad y = \sqrt{r^2 - x^2}$$

Substitute the numbers, with units (step 7):

$$y = \sqrt{(525 \text{ km})^2 - (4.50 \times 10^2 \text{ km})^2} = 2.70 \times 10^2 \text{ km}$$

**REMARKS** Note that the negative solution has been disregarded, because it’s not physically meaningful. In checking (step 8), note that the units are correct and that an approximate answer can be obtained by using the easier quantities, 500 km and 400 km. Doing so gives an answer of 300 km, which is approximately the same as our calculated answer of 270 km.

**QUESTION 1.11** What is the answer if both the distance traveled due east and the direct return distance are both doubled?

**EXERCISE 1.11** A plane flies 345 km due south, then turns and flies 615 km at a heading north of east, until it’s due east of its starting point. If the plane now turns and heads for home, how far will it have to go?

**ANSWER** 509 km

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**SUMMARY**

1.1 Standards of Length, Mass, and Time

The physical quantities in the study of mechanics can be expressed in terms of three fundamental quantities: length, mass, and time, which have the SI units meters (m), kilograms (kg), and seconds (s), respectively.

1.2 The Building Blocks of Matter

Matter is made of atoms, which in turn are made up of a relatively small nucleus of protons and neutrons within a cloud of electrons. Protons and neutrons are composed of still smaller particles, called quarks.

1.3 Dimensional Analysis

Dimensional analysis can be used to check equations and to assist in deriving them. When the dimensions on both sides of the equation agree, the equation is often correct up to a numerical factor. When the dimensions don’t agree, the equation must be wrong.
1.4 Uncertainty in Measurement and Significant Figures

No physical quantity can be determined with complete accuracy. The concept of significant figures affords a basic method of handling these uncertainties. A significant figure is a reliably known digit, other than a zero, used to locate the decimal point. The two rules of significant figures are as follows:

1. When multiplying or dividing using two or more quantities, the result should have the same number of significant figures as the quantity having the fewest significant figures.
2. When quantities are added or subtracted, the number of decimal places in the result should be the same as in the quantity with the fewest decimal places.

Use of scientific notation can avoid ambiguity in significant figures. In rounding, if the last digit dropped is less than 5, simply drop the digit; otherwise, raise the last retained digit by one.

1.5 Conversion of Units

Units in physics equations must always be consistent. In solving a physics problem, it’s best to start with consistent units, using the table of conversion factors on the front endsheets as necessary.

Converting units is a matter of multiplying the given quantity by a fraction, with one unit in the numerator and its equivalent in the other units in the denominator, arranged so the unwanted units in the given quantity are canceled out in favor of the desired units.

1.6 Estimates and Order-of-Magnitude Calculations

Sometimes it’s useful to find an approximate answer to a question, either because the math is difficult or because information is incomplete. A quick estimate can also be used to check a more detailed calculation. In an order-of-magnitude calculation, each value is replaced by the closest power of ten, which sometimes must be guessed or estimated when the value is unknown. The computation is then carried out. For quick estimates involving known values, each value can first be rounded to one significant figure.

1.7 Coordinate Systems

The Cartesian coordinate system consists of two perpendicular axes, usually called the x-axis and y-axis, with each axis labeled with all numbers from negative infinity to positive infinity. Points are located by specifying the x- and y-values. Polar coordinates consist of a radial coordinate r, which is the distance from the origin, and an angular coordinate θ, which is the angular displacement from the positive x-axis.

1.8 Trigonometry

The three most basic trigonometric functions of a right triangle are the sine, cosine, and tangent, defined as follows:

\[
\sin \theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{y}{r} \\
\cos \theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{x}{r} \\
\tan \theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{y}{x}
\]

[1.1]

The Pythagorean theorem is an important relationship between the lengths of the sides of a right triangle:

\[
r^2 = x^2 + y^2
\]

[1.2]

where r is the hypotenuse of the triangle and x and y are the other two sides.

MULTIPLE-CHOICE QUESTIONS

1. A rectangular airstrip measures 32.30 m by 210 m, with the width measured more accurately than the length. Find the area, taking into account significant figures. (a) 6.783 × 10^3 m^2 (b) 6.783 × 10^3 m^2 (c) 6.78 × 10^3 m^2 (d) 6.8 × 10^3 m^2 (e) 7 × 10^3 m^2

2. Suppose two quantities, A and B, have different dimensions. Determine which of the following arithmetic operations could be physically meaningful. (a) A + B (b) B − A (c) A − B (d) A/B (e) AB

3. Newton’s second law of motion (Chapter 4) says that the mass of an object times its acceleration is equal to the net force on the object. Which of the following gives the correct units for force? (a) kg · m/s^2 (b) kg · m^2/s^2 (c) kg/m · s^2 (d) kg · m/s (e) none of these

4. Use the rules for significant figures to find the answer to the addition problem 21.4 + 15 + 17.17 + 4.003. (a) 57.573 (b) 57.57 (c) 57.6 (d) 58 (e) 60

WebAssign The multiple-choice questions in this chapter may be assigned online in Enhanced WebAssign.